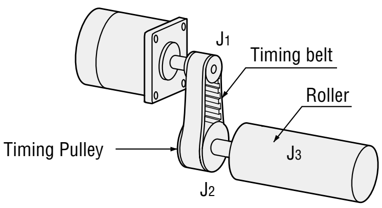
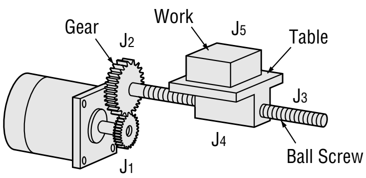
Basic Equations

1. Basic calculation formulas

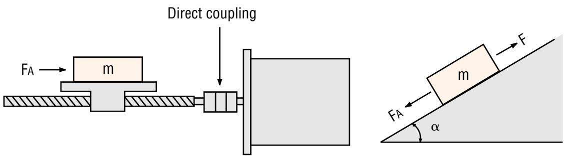
Drive mechanism \ Element	Resolution (minimum feed) step angle	Movement and number of pulses
Basic	$\Delta l = \Delta l_0 \frac{\theta s}{i}$ [mm/step] ……………①	$l = A \cdot \Delta l$ [mm] ……………⑥
Belt drive 	$\Delta l = \frac{\pi \cdot D}{360^\circ} \cdot \frac{\theta s}{i}$ [mm/step] ……………② $D = \frac{360^\circ \cdot \Delta l \cdot i}{\pi \ \theta s}$ [mm] ……………③	$l = v \cdot t$ [mm] ……………⑦ $A = \frac{l}{\Delta l}$ [pulse] ……………⑧
Ball screw 	$\Delta l = \frac{P_B}{360^\circ} \cdot \frac{\theta s}{i}$ [mm/step] ……………④ $P_B = \frac{360^\circ \cdot \Delta l \cdot i}{\theta s}$ [mm/rev] ……………⑤	$A = f \cdot t$ [pulse] ……………⑨

Δl = Resolution (minimum feed) [mm/step]
Δl_0 = Unit of movement at final stage [mm/°]
θs = Step angle [°/step]
i = Gear ratio
P_B = Lead pitch [mm/rev]
v = Movement speed [mm/s]
f = Pulse speed [Hz]
D = Final pulley diameter [mm]
A = Number of pulses [pulse]
l = Movement [cm]
t = Positioning period [s]

Speed and pulse speed	Speed at final stage and pulse speed	Total inertia seen from the motor
$\nu = \Delta l \cdot f$ [mm/s] ······⑩ $f = \dfrac{\nu}{\Delta l}$ [Hz] ······⑪		J_L : Total inertia in motor shaft equivalents J_n : Inertia of parts
$\nu = \dfrac{\pi D}{360^\circ} \cdot \dfrac{\theta_s}{i} \cdot f$ [mm/s] ······⑫ $f = \dfrac{360^\circ \cdot i \cdot \nu}{\pi D \cdot \theta_s}$ [Hz] ······⑬	$N = \dfrac{\theta_s \cdot f}{6 \cdot i}$ [r/min] ······⑯ $f = \dfrac{6 \cdot i \cdot N}{\theta_s}$ [Hz] ······⑰	$J_L = J_1 + \dfrac{J_2 + J_3}{i^2}$ [kg·m²] ······⑱
$\nu = \dfrac{P_B}{360^\circ} \cdot \dfrac{\theta_s}{i} \cdot f$ [mm/s] ······⑭ $f = \dfrac{360^\circ \cdot i \cdot \nu}{P_B \cdot \theta_s}$ [Hz] ······⑮		$J_L = J_1 + \dfrac{J_2 + J_3 + J_4 + J_5}{i^2}$ [kg·m²] ······⑲

2. Formulas for load torque

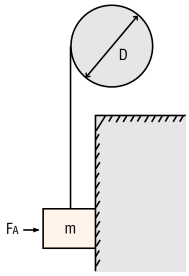
Ball screw



$$T_L = \left(\frac{FP_B}{2\pi\eta} + \frac{\mu_0 F_0 P_B}{2\pi}\right) \times \frac{1}{i} \text{ [N·m]} \quad \text{......(20)}$$

$$F = F_A + mg(\sin\alpha + \mu\cos\alpha) \text{ [N]} \quad \text{......(21)}$$

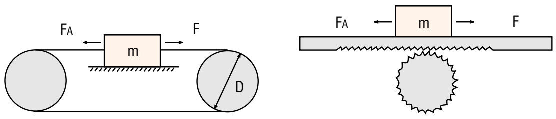
Pulley



$$T_L = \frac{\mu F_A + mg}{2\pi} \cdot \frac{\pi D}{i}$$

$$= \frac{(\mu F_A + mg)D}{2i} \text{ [N·m]} \quad \text{......(22)}$$

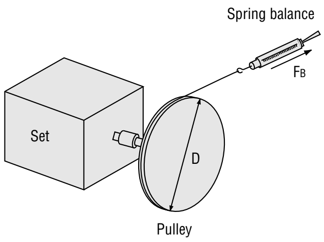
Wire belt drive, rack and pinion drive



$$T_L = \frac{F}{2\pi\eta} \cdot \frac{\pi D}{i} = \frac{FD}{2\eta i} \text{ [N·m]} \quad \text{......(23)}$$

$$F = F_A + mg(\sin\alpha + \mu\cos\alpha) \text{ [N]} \quad \text{......(24)}$$

By actual measurement

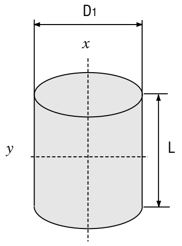


$$T_L = \frac{F_B D}{2} \text{ [N·m]} \quad \text{......(25)}$$

F = Load in shaft direction [N]
F_0 = Pilot pressure Load [N] (≒ 1/3 F)
μ_0 = Internal friction coefficient of pilot pressure nut (0.1 to 0.3)
η = Efficiency (0.85 to 0.95)
i = Gear ratio
P_B = Ball screw pitch [m/rev]
F_A = External force [N]
F_B = Force when main shaft begins to rotate [N]
m = Total mass of work and table [kg]
μ = Frictional coefficient of sliding surfaces (0.05)
α = Angle of inclination [˚]
D = Final pulley diameter [m]
g = 9.8 [m/s^2]

3. Formulas for calculating inertial moments

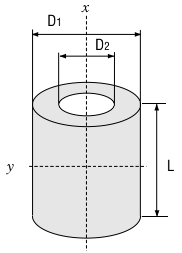
Inertia of a cylinder



$$J_x = \frac{1}{8} m D_1^2 = \frac{\pi}{32} \rho L D_1^4 \text{ [kg·m}^2\text{]} \quad (26)$$

$$J_y = \frac{1}{4} m \left(\frac{D_1^2}{4} + \frac{L^2}{3} \right) \text{ [kg·m}^2\text{]} \quad (27)$$

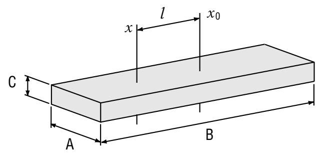
Inertia of a hollow cylinder



$$J_x = \frac{1}{8} m \left(D_1^2 + D_2^2 \right) = \frac{\pi}{32} \rho L \left(D_1^4 - D_2^4 \right) \text{ [kg·m}^2\text{]} \quad (28)$$

$$J_y = \frac{1}{4} m \left(\frac{D_1^2 + D_2^2}{4} + \frac{L^2}{3} \right) \text{ [kg·m}^2\text{]} \quad (29)$$

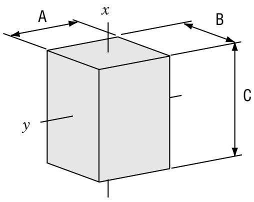
Inertia for off centered axis of rotation



l = Distance between x and x_0 axes [in.]

$$J_x = J_{x0} + m\, l^2 = \frac{1}{12} m \left(A^2 + B^2 + 12\, l^2 \right) \text{ [kg·m}^2\text{]} \quad (30)$$

Inertia of a rectangular pillar



$$J_x = \frac{1}{12} m \left(A^2 + B^2 \right) = \frac{1}{12} \rho A B C \left(A^2 + B^2 \right) \text{ [kg·m}^2\text{]} \quad (31)$$

$$J_y = \frac{1}{12} m \left(B^2 + C^2 \right) = \frac{1}{12} \rho A B C \left(B^2 + C^2 \right) \text{ [kg·m}^2\text{]} \quad (32)$$

Inertia of an object in linear motion

$$J = m \left(\frac{v}{\omega} \right)^2 = m \left(\frac{A}{2\pi} \right)^2 \text{ [kg·m}^2\text{]} \quad (33)$$

A = Unit of movement [m/rev]

Density		
Iron	$\rho = 7.9 \times 10^3$ [kg/m³]	
Aluminum	$\rho = 2.8 \times 10^3$ [kg/m³]	
Bronze	$\rho = 8.5 \times 10^3$ [kg/m³]	
Nylon	$\rho = 1.1 \times 10^3$ [kg/m³]	

J_x = Inertia on x axis [kg·m²]
J_y = Inertia on y axis [kg·m²]
J_0 = Inertia on x_0 axis (passing through center of gravity) [kg·m²]
m = Mass [kg]
D_1 = External diameter [m]
D_2 = Internal diameter [m]
ρ = Density [kg / m³]
L = Length [m]